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Columbia University
in the City of New York

DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

IMPACT ON THE SURFACE OF A COMPRESSIBLE FLUID

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Office of Naval Research
Project NR 064-428
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ABSTRACT

The pressure field generated by the impact of a rigid or flexible body on the surface of an inviscid, compressible fluid is considered. Some general results for the forces generated by the impact are given. Pressure distributions for rigid wedges are computed from known analagous solutions of linearized supersonic airfoil theory. In the impact problem, the transition from compressible to incompressible behavior is demonstrated as the velocity of impact is reduced.

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TABLE OF CONTENTS

LIST OF FIGURES

1. Introduction

The impact of a body onto the free surface of a fluid is of interest in the landing of seaplanes, slamming of ships, and the entry of a missile or other projectile intc a body of water. When the impact velocity is sufficiently large and/or the body is sufficiently blunt, it is necessary to take account of the compressibility of the fluid to obtain realistic results. Otherwise, the fluid may be considered incompressible.

The incompressible case has been extensively treated both theoretically and experimentally [1], [2], [3], [4]. The compressible phase has received less attention and, in some cases, rather rough approximations of the pressure field are suggested [5], [6], [7], [8]. A summary of previous work is given in Ref. [9].

The purposes of the present report are: (a) to give some general results for the force developed during the compressible phase, (b) call attention to a known but little used analogy of the impact problem to linearized supersonic flow, (c) present numerical results for wedges which may be computed directly from supersonic flow solutions. These computations supply substantial corrections to the work of Trilling reported in [5], for example. In addition, the numerical results demonstrate the transition from compressible to incompressible behavior of the fluid which has not been well defined heretofore.

2. Formulation of the Impact Problem

The general problem considered is the impact of a rigid or a flexible body onto the surface of a semi-infinite fluid as shown in Fig. 1. The velocity of the body V_o is assumed to be normal to the surface of the fluid at the instant of impact. It is also assumed that V_o is small compared to c, the velocity of sound in the fluid, i.e., the entry Mach number $M = V_o/c$ is much less than unity. The fluid is considered to be a slightly compressible liquid like water so that the change in the density ρ due to the impact is small compared to the initial density ρ_o.

It might be expected that under the conditions $V_o/c \ll 1$ and $\rho/\rho_o - 1 \ll 1$ that the fluid could be treated as incompressible. This is correct if the body is not too blunt. For a blunt body, the area of contact between the fluid and the solid expands rapidly. The velocity, V_e, of the boundary of the contact area is of the order $V_o/\tan \alpha$ where α is the typical slope of the body, Fig. 1. If V_e is larger than c, the action of the body on the fluid corresponds to a loaded area which expands supersonically with respect to the fluid. In this case, the compressibility of the fluid must be taken into account to obtain realistic results. Defining an edge Mach number $M_e = V_e/c$, it may be anticipated that the fluid must be treated as compressible whenever M_e is of the order of 1 or more.

The two conditions, $V_o/c \ll 1$ and $V_e/c = 0(1)$ are simultaneously satisfied only if $\tan \alpha \ll 1$. Hence α will be small and the penetration of the body into the fluid is small whenever compressibility must be taken into account. This fact together with the assumption of small changes in density justifies the use of the acoustic equations for the fluid. Further, it is the basis for linearization of the boundary conditions: the pressure exerted on the fluid by the body is considered to be applied on the plane of the initial free surface, $z = 0$ and the z component of velocity of the fluid at $z = 0$ is set equal to the velocity of the body.

The fluid being assumed inviscid and at rest initially, the motion of the fluid during impact is irrotational and a velocity potential φ exists such that the velocity $\underline{u}$ is

$$\underline{u} = \nabla\varphi \tag{1}$$

where φ satisfies the wave equation

$$\nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \tag{2}$$

Within the present approximation, the pressure is given by

$$p = -\rho_o \frac{\partial\phi}{\partial t} \tag{3}$$

The initial conditions of no motion and zero pressure in the fluid are:

$$\left.\begin{aligned} \phi &= 0 \\ \frac{\partial\phi}{\partial t} &= 0 \end{aligned}\right\} \text{ at } t = 0 \tag{4}$$

The boundary conditions to be applied on the surface $z = 0$ are

$$\left.\begin{aligned} \frac{\partial\varphi}{\partial z} &= V \quad \text{on} \quad A_o \\ \varphi &= 0 \quad \text{on} \quad \bar{A}_o \end{aligned}\right\} \quad t > 0 \qquad (5)$$

where A_o is the area of the intersection of the plane $z = 0$ and the body at any time t ; $\bar{A}_o$ is remainder of plane $z = 0$, A_o excluded; V is the z component of the velocity of the body. V will be a function of (x,y,t) for a flexible body and of t only for a symmetric rigid body.

The equations governing the motion and deformation of the impacting body must also be considered in general. Such equations together with (1) through (5) complete the formulation of the impact problem.

3. The Impact Force

A general formula may be derived for the total impact force in terms of the velocities on the surface $z = 0$. The final formula is directly useful where the edge Mach number is supersonic.

The velocity potential in any case may be written in terms of a retarded potential [5], [10]:

$$\phi(x,y,z,t) = -\frac{1}{2\pi} \iint \frac{\phi_z(\xi,\eta,o,t - \frac{r_1}{c})}{r_1} \, d\xi d\eta \qquad (6)$$

where $\quad r_1 = \sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}$

and $\phi_z(x,y,o,t)$ is the z component of velocity on the surface $z = 0$. The value of ϕ is different from zero only over the area of contact A_o for a supersonic edge Mach number, $M_e > 1$ (Fig. 2a). For $M_e < 1$, ϕ_z has non-zero values over A_o where it matches the velocity of the body and also over the area A_1 which is the circle $r = ct$ minus the area A_o (Fig. 2b). The pressure over A_1 is zero but the fluid will usually move upwards in this zone. The limits of the integration in (6) may be considered to extend over the entire ξ, η plane since $\phi_z(x,y,o,t)$ is zero for $t < 0$. Then the pressure is

$$p = -\rho \frac{\partial\phi}{\partial t} = \frac{\rho}{2\pi} \iint \frac{\phi_z'(\xi,\eta,o,t - \frac{r_1}{c})}{r_1} d\xi d\eta \tag{7}$$

where the prime denotes differentiation with respect to $\tau = t - r_1/c$.

The force exerted by the body on the fluid is

$$F(t) = \iint_{A_o} p(x,y,o,t)dxdy = \frac{\rho}{2\pi} \iint_{A_o} dxdy \iint \frac{\phi_z'(\xi,\eta,o,t - \frac{r}{c})}{r} d\xi d\eta \tag{8}$$

where $\quad r = \sqrt{(x - \xi)^2 + (y - \eta)^2}$. In both cases, $M_e \gtrless 1$, the pressure on the surface $z = 0$ is zero outside of the area of contact, A_o. Hence the integration over A_o

in (8) may be extended over the full plane $z = 0$ and then the order of integrations may be reversed to give

$$F(t) = \frac{\rho}{2\pi} \iint d\xi d\eta \iint \frac{\phi_z'(\xi,\eta,o,t - \frac{r}{c})}{r} dxdy \tag{9}$$

The variables of the x, y integration may be changed to r, θ where

$$\theta = \arc\tan \frac{y - \eta}{x - \xi} \tag{10}$$

The integrand in (9) is independent of θ. Hence using $dxdy = rdrd\theta$ and integrating with respect to θ gives

$$F(t) = \rho \iint d\xi d\eta \int_o^{ct} \phi_z'(\xi,\eta,o,t - \frac{r}{c})dr \tag{11}$$

where the limits on r have been restored to reflect the fact that $\phi_z(x,y,o,t - r/c) = 0$, for $(t - r/c) < 0$.

The prime derivative in (11) is with respect to $\tau = t - r/c$. Using τ as a variable of integration the inner integral in (11) may be directly evaluated. The result is:

$$F(t) = \rho c \iint \phi_z(\xi,\eta,o,t)d\xi d\eta \tag{12}$$

where the integration is carried out over the entire plane $z = 0$. The most interesting aspect of the formula (12) is that the force $F(t)$ is expressed entirely in terms of the instantaneous surface velocities at the time t. The detailed pressure distribution and the motion of the fluid at any time t depend on the values of the surface velocities at all previous times as well. Examples are given in the succeeding sections.

For the case $M_e > 1$, the integrand in (12) is zero outside of A_o, Fig. 2a, and

$$F(t) = \rho c \overline{V}_o A_o \tag{13}$$

where $\overline{V}_o$ is the mean velocity of the body over the area of contact A_o. For a rigid body, $\overline{V}_o$ is equal to the instantaneous velocity of the body.

The force given by (13) is exactly the same as that which results from a one-dimensional motion in which a piston of area A_o is forced with velocity $\overline{V}_o$ into a rigid-walled cylinder filled with fluid and of cross section A_o. In this one-dimensional case, the pressure is uniform and equal to $\rho c \overline{V}_o$. In the impact case under consideration the pressure distribution may be far from uniform, but its mean value is $\rho c \overline{V}_o$.

For an impact with a subsonic edge Mach number, $M_e < 1$, $\phi_z(x,y,o,t)$ is zero outside of A_1, Fig. 2b, and

$$F = \rho c \overline{V}_o A_o + \rho c \overline{V}_1 A_1 \tag{14}$$

where $\overline{V}_o$ and $\overline{V}_1$ are the mean velocities over the areas A_o and A_1 respectively. Ordinarily $\overline{V}_1$ will be negative. The term $\rho c \overline{V}_1 A_1$ represents the effect of relief of pressures on the body due to the presence of the free surface of the fluid.

The following alternative derivation of (12) gives additional insight into the physical effects involved.

Consider the solution of (2) corresponding to a point source of constant strength at $r = 0$, $z = 0$ starting at $t = 0$. The velocity potential is

$$\phi = -\frac{B}{2\pi r_1} H(t - \frac{r_1}{c}) \qquad (15)$$

where H is the Heaviside unit step function and B is the constant strength. The pressure given by (3) is

$$p = \frac{\rho_o cB}{2\pi r_1} \delta(t - \frac{r_1}{c}) \qquad (16)$$

and the z component of velocity given by (1) is

$$\phi_z = \frac{Bz}{2\pi r_1^2} [\frac{1}{c} \delta(t - \frac{r_1}{c}) + \frac{1}{r_1} H(t - \frac{r_1}{c})] \qquad (17)$$

where δ is the Dirac delta function. The value of $\phi_z = (x,y,o,t)$ is zero except for the point $r = 0$. Considering the limit as $z \to 0$, it may be shown that

$$\iint \phi_z(x,y,o,t)dxdy = BH(t) \qquad (18)$$

Hence

$$\phi_z(x,y,o,t) = \frac{B}{2\pi r} \delta(r)H(t) \qquad (19)$$

The pressure (16) may also be derived by solving (2) for ϕ considering (19) as a prescribed boundary condition and using a Laplace transform in time, a cosine transform in z and a Hankel transform in r .

The pressure distribution (17) gives rise to a force on the plane $z = 0$ which is

$$f(t) = \frac{\rho_o B}{2\pi} \int_0^\infty \int_0^{2\pi} \frac{\delta(t - \frac{r}{c})}{r} r dr d\theta = \rho c B H(t) \tag{20}$$

The fact that $f(t)$ is constant in time is the essential reason that $F(t)$ given by (13) depends only on the instantaneous velocities of the surface.

If B is variable with time but is zero for $t < 0$, the force $f(t)$ on the plane $z = 0$ may be written using Eq. (20) and Duhamel's formula. Thus

$$f(t) = \rho c \int_0^t \frac{dB}{dt} dt = \rho c B(t) \tag{21}$$

which shows that the force depends only on the instantaneous source strength.

In the impact problem, the force on the entire $z = 0$ plane may be derived by integrating (21) over the plane where the source strength B is equal to $\phi_z(x,y,o,t)dA$ by virtue of (18). Hence the total force on $z = 0$ in the impact problem is

$$F(t) = \iint \rho c \phi_z(x,y,o,t) dxdy \tag{22}$$

which is identical to (13) The pressure outside the contact area A_o , Fig. 2, is zero for both $M_e > 1$ and $M_e < 1$ so (22) yields, in fact, the force exerted by the body on the fluid.

4. The Analogy to Linearized Supersonic Flow

When the impacting body is a cylinder of arbitrary cross section, whose generators are parallel to the surface of the undisturbed fluid, there exists an analogy between the impact problem and a problem in linearized supersonic flow theory. In such an impact the pressure and velocity fields are two-dimensional, i.e., they are functions of (x,z,t) when the y axis is taken parallel to the generators of the cylindrical body. The formulation of the impact problem is still given by Eqs. (1) through (5).

The analogous problem in linearized supersonic flow theory is that of a plane lifting surface placed in a supersonic stream at a small angle of attack. The disturbance of the free stream due to the body may be described by a perturbation velocity field which has a velocity potential $\phi'(x',y',z')$ where (x',y',z') are the spatial coordinates of the system. $\phi'(x',y',z')$ satisfies the wave equation

$$\frac{\partial^2\phi'}{\partial x'^2} + \frac{\partial^2\phi'}{\partial y'^2} = B^2 \frac{\partial^2\phi'}{\partial z'^2} \qquad (23)$$

where $B^2 = U^2/c^2 - 1$, U is the supersonic stream velocity which is taken to be in the z' direction and c is the sound speed in the fluid. For a lifting surface at a small angle of attack the boundary conditions for linearized theory are

$$\frac{\partial\phi'}{\partial y'}(x',o,z') = U\alpha(x',z') \qquad \text{on } A_o'$$
$$\phi'(x',o,z') = 0 \qquad \text{on } \bar{A}_o' \tag{24}$$

where A_o' is the projection of the body on the plane $y' = 0$, and $\bar{A}_o'$ is the remainder of the plane $y' = 0$.

There is an extensive literature, e.g., [10], [11] and [12], which treats this linearized supersonic flow problem. Comparing Eqs. (23) and (24) to Eqs. (1) through (5) it is seen that with the correspondence

$$\phi' \to \phi \;,\; x' \to x \;,\; y' \to z \;,\; z' \to t$$
$$B^2 \to \frac{1}{c^2} \;,\; A_o' \to A_o \;,\; \bar{A}_o' \to \bar{A}_o \tag{25}$$

the supersonic flow problem is exactly the same as the impact problem. Hence the solution of any particular impact problem is obtained immediately if the corresponding supersonic flow case is available and vice versa.

5. Impact of a Rigid Wedge

The impact problem for a rigid wedge of arbitrary angle is a case for which the analogous solutions are available in the supersonic flow literature. These cases are treated by the theory of conical fields [10].

Consider a wedge of semi-vertex angle β which moves in the z direction with a constant velocity V. The geometry of the problem at the instant of impact is shown in Fig. 3.

The penetration of the body into the fluid being assumed small, the boundary conditions (5) are

$$\begin{aligned} \frac{\partial \Phi}{\partial z}(x,o,t) &= V \qquad Vt\ \mathrm{ctn}\ (\alpha+2\beta) < x < Vt\ \mathrm{ctn}\ \alpha \\ \Phi(x,o,t) &= 0 \qquad Vt\ \mathrm{ctn}\ \alpha < x\ ,\ x < Vt\ \mathrm{ctn}(\alpha+2\beta) \end{aligned} \tag{26}$$

The width of the wetted surface of the wedge as a function of time is shown in the (x,t) plane as the shaded region of Fig. 4. The lines

$$\begin{aligned} x_1 &= Vt\ \mathrm{ctn}\ \alpha \\ x_2 &= Vt\ \mathrm{ctn}\ (\alpha + 2\beta) \end{aligned} \tag{27}$$

are the traces of the intersection of the wedge and the free surface. The velocities of these intersections are

$$\begin{aligned} \dot{x}_1 &= V\ \mathrm{ctn}\ \alpha \\ \dot{x}_2 &= V\ \mathrm{ctn}\ (\alpha + 2\beta) \end{aligned} \tag{28}$$

and edge Mach numbers are defined by

$$M_1 = \frac{V}{c} \operatorname{ctn} \alpha$$
$$M_2 = V/c \mid \operatorname{ctn} (\alpha + 2\beta) \mid \tag{29}$$

These edge Mach numbers will be called supersonic or subsonic depending on whether they are greater or less than 1. Three possibilities now arise:

(a) Both edge Mach numbers are supersonic.

(b) Both edge Mach numbers are subsonic.

(c) The mixed case.

Equation (2) is hyperbolic and there exists a real characteristic cone in x,z,t space given by

$$ct - \sqrt{x^2 + z^2} = 0 \tag{30}$$

The intersection of this characteristic cone with the plane $z = 0$ is

$$x = \pm ct \tag{31}$$

and the case of supersonic and subsonic edge Mach numbers correspond to the traces of the wetted surface in Fig. 4 lying outside or inside of the characteristic cone. The analogues of all three cases (a), (b), and (c) in supersonic flow have been treated by Ward [10] and Lagerstrom [11].

(a) Both edge Mach numbers supersonic.

In this case the projection of the wetted surface of the wedge intersects the characteristic cone as shown in Fig. 5.

Under the transformation

$$\frac{r}{ct} = x_1 = r_1 \cos\theta$$
$$\frac{z}{ct} = z_1 = r_1 \sin\theta \qquad (32)$$

the wave equation (2) becomes

$$r_1^2(r_1^2-1)\frac{\partial^2\phi}{\partial r_1^2} + r_1(2r_1^2-1)\frac{\partial\phi}{\partial r_1} - \frac{\partial^2\phi}{\partial\theta^2} = 0 \qquad (33)$$

This equation is elliptic in the region $r_1 < 1$, and hyperbolic in the region $r_1 > 1$. The regions of interest are shown in Fig. 6 where the solution is extended anti-symmetrically to the entire x_1, z_1 plane. The region outside the shaded region and the circle $r_1 = 1$, is the region which has not yet felt the disturbance due to the entering wedge.

The pressure in the hyperbolic (shaded) region can be determined by simple momentum considerations or by use of the differential equation in this region as in [10]. The pressure is then known on the boundary of the elliptic region. Equation (33) is reduced to Laplace's equation by Chaplygin's transformation of the radial coordinate

$$r_1 = \frac{2S}{1+S^2} \qquad (34)$$

It is then possible to solve for the pressure distribution throughout the region in which the equation is elliptic by analytic function theory. On the wedge surface $z = 0$, the pressure is given by

$$\frac{p(x,o,t)}{\rho Vc} = \frac{M_2}{\sqrt{M_2^{\,2}-1}} \,, \quad Vt \text{ ctn } (\alpha+2\beta) < x < -ct \qquad (35)$$

$$\frac{p(x,o,t)}{\rho Vc} = \frac{2}{\pi}\left[\frac{M_2}{\sqrt{M_2^{\,2}-1}} \tan^{-1}\sqrt{\frac{(M_2-1)(ct-x)}{(M_2+1)(ct+x)}} + \frac{M_1}{\sqrt{M_1^{\,2}-1}} \tan^{-1}\sqrt{\frac{(M_1-1)(ct+x)}{(M_1+1)(ct-x)}}\right] , \quad -ct < x < ct$$

(36)

$$\frac{p(x,o,t)}{\rho Vc} = \frac{M_1}{\sqrt{M_1^{\,2}-1}} \,, \quad ct < x < Vt \text{ ctn } \alpha \qquad (37)$$

In the symmetric wedge entry case $\text{ctn } \alpha = |\text{ctn } (\alpha+2\beta)|$ and the above expressions become

$$\frac{p(x,o,t)}{\rho Vc} = \frac{M_o}{\sqrt{M_o^{\,2}-1}} \,; \quad -Vt \tan\beta < x < -ct \,, \quad ct < x < Vt \tan\beta$$

(38)

$$\frac{p(x,o,t)}{\rho Vc} = \frac{2}{\pi}\frac{M_o}{\sqrt{M_o^{\,2}-1}}\tan^{-1}\sqrt{\frac{M_o^{\,2}-1}{1-\left(\frac{xM_o}{Vt\tan\beta}\right)^2}}\;;\; -ct < x < ct \tag{39}$$

where $M_o = M_1 = M_2$ = edge Mach number.

The total force acting on the wedge as a function of time can be obtained by integrating the pressure over the instantaneous wetted surface on which it acts. The result is

$$F(t) = \rho Vc \cdot Vt\left[\operatorname{ctn}\alpha + |\operatorname{ctn}(\alpha + 2\beta)|\right] \tag{40}$$

This demonstrates the validity of Eq. (13) where for the present case $A_o = Vt\left[\operatorname{ctn}\alpha + |\operatorname{ctn}(\alpha + 2\beta)|\right]$.

(b) Both edge Mach numbers subsonic.

The transformation (32) is used again and the wave equation (2) reduces to (33). The domain of dependence of the impacting wedge now lies entirely within the characteristic cone (30) or in terms of the reduced coordinates entirely within the circle $x_1^{\,2} + z_1^{\,2} = 1$, Fig. 7. Within this region (33) is elliptic and the Chaplygin transformation of the radial coordinate (34) again transforms (33) into Laplace's equation. The solution is continued into the upper half of the circle antisymmetrically. Outside of and on the circle $r_1 = 1$ the pressure as well as the velocity components vanish. On the portion of the x_1 axis, $M\operatorname{ctn}(\alpha + 2\beta) < x_1 < M\operatorname{ctn}\alpha$, (Fig. 7) corresponding to the trace of the wedge wetted surface the boundary condition on the normal velocity is

$$\frac{\partial\phi}{\partial z_1}(x_1, 0) = V \tag{41}$$

This problem corresponds to a plane subsonic wing at an angle of attack, the solution of which is given by Ward [10]. The resulting expression for the pressure on the wedge surface $z = 0$ is

$$\frac{p(x,o,t)}{\rho Vc} = \frac{1}{2E - (1-k^2)K}\,\frac{1}{\sqrt{(1+M_1)(1+M_2)}}\left[M_1\sqrt{\frac{\frac{x}{ct}+M_2}{M_1-\frac{x}{ct}}} + M_2\sqrt{\frac{M_1-\frac{x}{ct}}{\frac{x}{ct}+M_2}}\right] \tag{42}$$

$$\text{for} \quad Vt \operatorname{ctn}(\alpha+2\beta) < x < Vt \operatorname{ctn}\alpha$$

where K and E are respectively the complete elliptic integrals of the first and second kind of modulus k and

$$k^2 = \frac{(1-M_1)(1-M_2)}{(1+M_1)(1+M_2)} \tag{43}$$

For a symmetric wedge, (42) simplifies to

$$\frac{p(x,o,t)}{\rho Vc} = \frac{1}{E_1}\,\frac{M_o}{\sqrt{1-\left(\frac{x}{Vt \operatorname{ctn}\alpha}\right)^2}} \tag{44}$$

where E_1 is the complete integral of the second kind with modulus k_1 and

$$k_1^2 = 1 - \left(\frac{1-k}{1+k}\right)^2 \tag{45}$$

For the symmetric case

$$k_1^2 = 1 - M_o^2 \tag{46}$$

The pressure field (42) can be integrated to give the total force acting on the wedge. The result is

$$F(t) = \rho Vc \frac{2\pi M_1 M_o}{\sqrt{(1+M_1)(1+M_2)} + \sqrt{(1-M_1)(1-M_2)}} \frac{ct}{E_1} \tag{47}$$

For the symmetric case the above expression simplifies and becomes

$$F(t) = \rho Vc \frac{\pi M_o}{2E_1} (2Vt \operatorname{ctn} \alpha) \tag{48}$$

It can be shown that $\pi M_o/2E_1 < 1$ so that the validity of (14) is demonstrated for this subsonic case.

(c) The mixed case.

In this case the projection on the $z = 0$ plane of one side of the wetted surface of the wedge intersects the characteristic cone (30) while the other side does not. The situation is shown in the reduced coordinates plane (x_1, z_1) in Fig. 8.

The same transformations and similar techniques as discussed above are applied. The solution is given by

Ward [10] and the pressure on the wedge surface $z = 0$ is

$$\frac{p(x,o,t)}{\rho Vc} = \frac{2}{\pi}\left[\frac{M_2}{1+M_2}\sqrt{\frac{(M_1+M_2)(ct-x)}{(M_1+1)(x+M_2 ct)}} + \frac{M_1}{\sqrt{M_1^2-1}}\tan^{-1}\sqrt{\frac{(M_1-1)(x+M_2 ct)}{(M_1+M_2)(ct-x)}}\right] \quad (49)$$

for $Vt \operatorname{ctn}(\alpha+2\beta) < x < ct$

$$\frac{p(x,o,t)}{\rho Vc} = \frac{M_1}{\sqrt{M_1^2-1}} \quad \text{for} \quad ct < x < Vt \operatorname{ctn} \alpha \quad (50)$$

Numerical results for a symmetric wedge entry are shown in Fig. 8, and for a nonsymmetric case in Fig. 9. The pressures are divided by ρVc, the pressure that would be developed in a one-dimensional impact case. The lengths are divided by the instantaneous width of the longer wetted side of the impacting wedge. Each curve represents a different edge Mach number. In Fig. 9 the ratio of the two edge Mach numbers for each unsymmetric wedge is $\mu = M_2/M_1 = 0.5$.

For supersonic edge Mach numbers the pressures are continuous. However, at the boundary of the elliptic hyperbolic regions (Fig. 6) there is a discontinuity in the derivative of the pressure. In the subsonic cases the pressure fields have a singularity at the wedge free surface intersection which is integrable and yields finite forces acting on the wedge.

A significant feature of the results for supersonic edge Mach numbers is that the pressure distribution is not uniform, the nonuniformity increasing as the edge Mach numbers decay to 1. This is in direct contrast to the uniform pressures assumed in [5], for example.

6. Impact of a Wedge on an Incompressible fluid.

The impact of a symmetric wedge onto the surface of an incompressible fluid was first treated by von Karman [1] and was developed further by Wagner [2]. The approach used corresponds exactly to the formulation given by (1) through (5) with c approaching infinity so that (2) becomes Laplace's equation. The boundary conditions (5) are again expressed by (26). The solution for an incompressible fluid turns out to be the same as one half of the flow due to a flat plate moving broadside through a fluid otherwise at rest, as suggested by von Karman. The growth of the wetted surface of the wedge with time results effectively in the width of the flat plate expanding at the same rate. The pressure distribution on the wedge, given in [2] and [3], is:

$$p(x,o,t) = \frac{\rho V^2 \operatorname{ctn} \alpha}{\sqrt{1 - \left(\frac{x}{Vt \operatorname{ctn} \alpha}\right)^2}} \tag{51}$$

and the force on the wedge is:

$$F(t) = \rho \pi V^3 t \operatorname{ctn}^2 \alpha \tag{52}$$

where all symbols have the same meaning as in the previous section.

It may be shown that the limit of the force for $c \to \infty$ in the subsonic case given by (48) is equal to (52). Further, the pressure distributions for the edge Mach number 0.5, 0.25 and 0 (incompressible case) as given by (44) and (51) are compared in Fig. 11 for fixed values of the impact velocity, wedge angle and fluid density; the sound speed c being variable. These curves imply that if the edge Mach numbers are less than 0.25, the effects of compressibility may be neglected.

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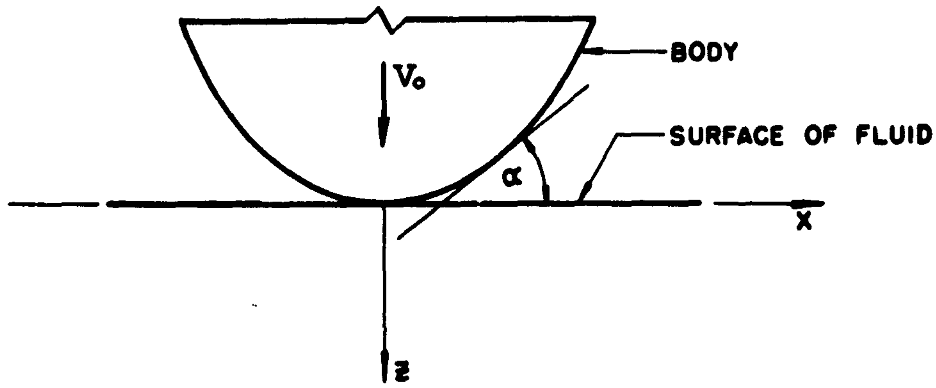
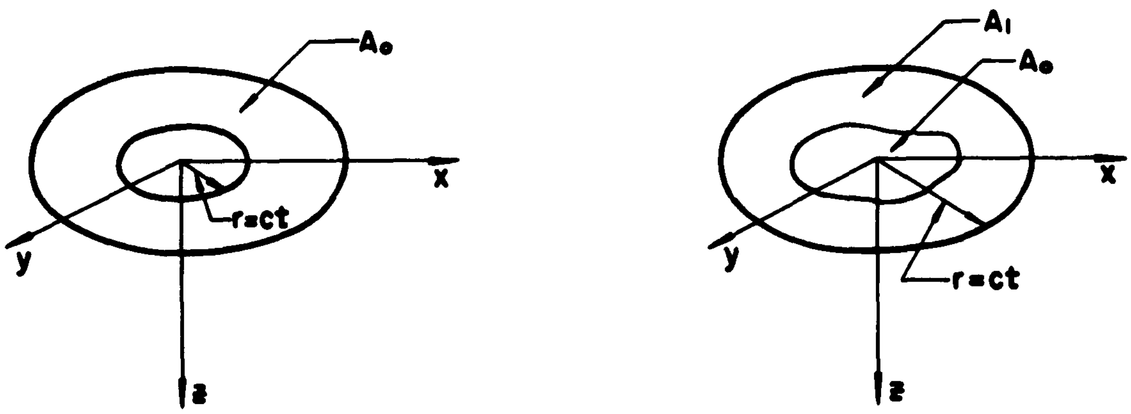


F I G . I CONDITIONS AT INSTANT OF IMPACT



(a) SUPERSONIC EDGE MACH NUMBER

(b) SUBSONIC EDGE MACH NUMBER

F I G . 2 CONDITIONS AT TIME t AFTER IMPACT

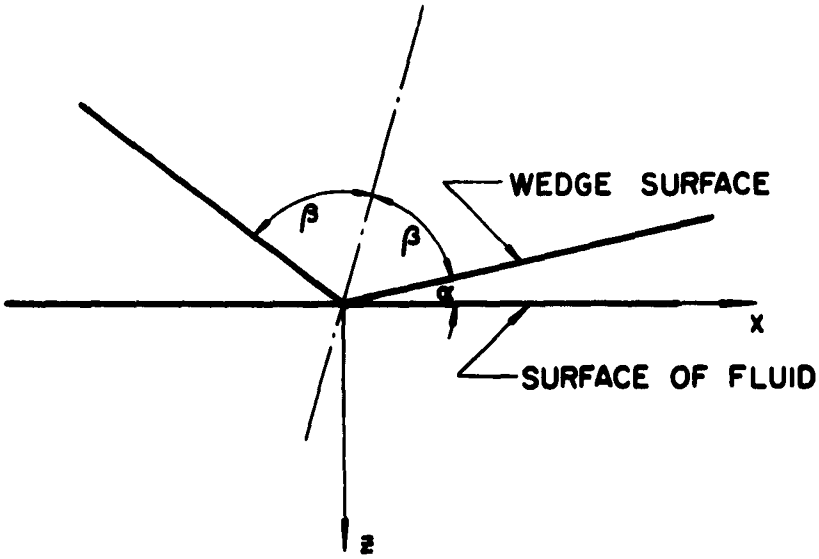


FIG. 3 CONFIGURATION OF WEDGE IMPACT PROBLEM AT INSTANT OF IMPACT

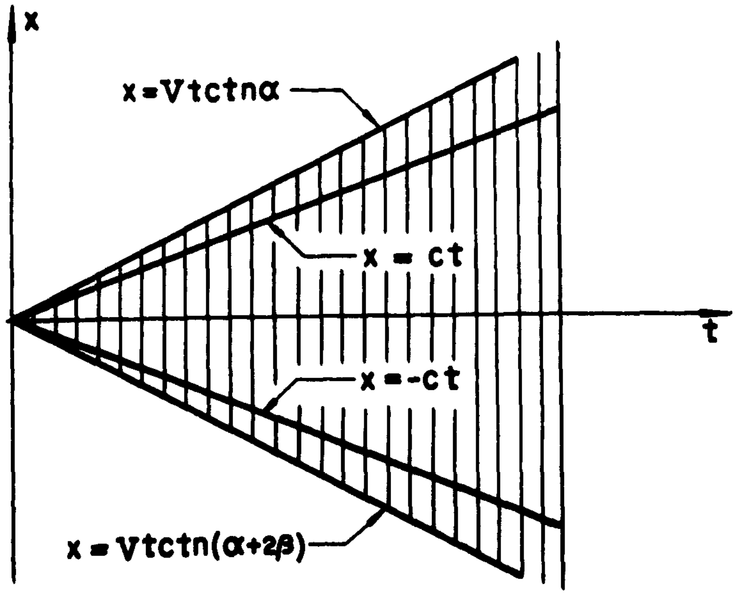


FIG. 4 WETTED SURFACE OF WEDGE IN (x, t) PLANE

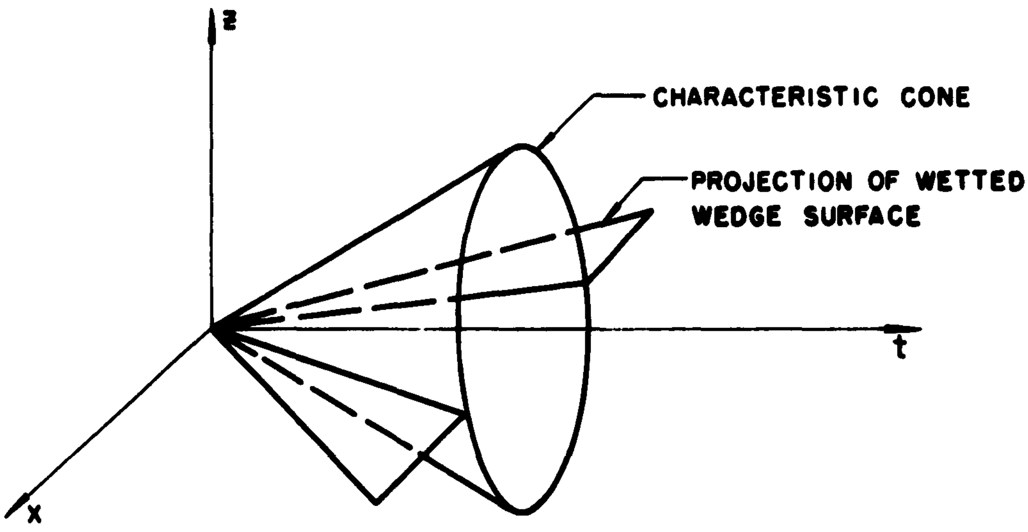


FIG. 5 INTERSECTION OF WETTED SURFACE OF WEDGE AND CHARACTERISTIC CONE

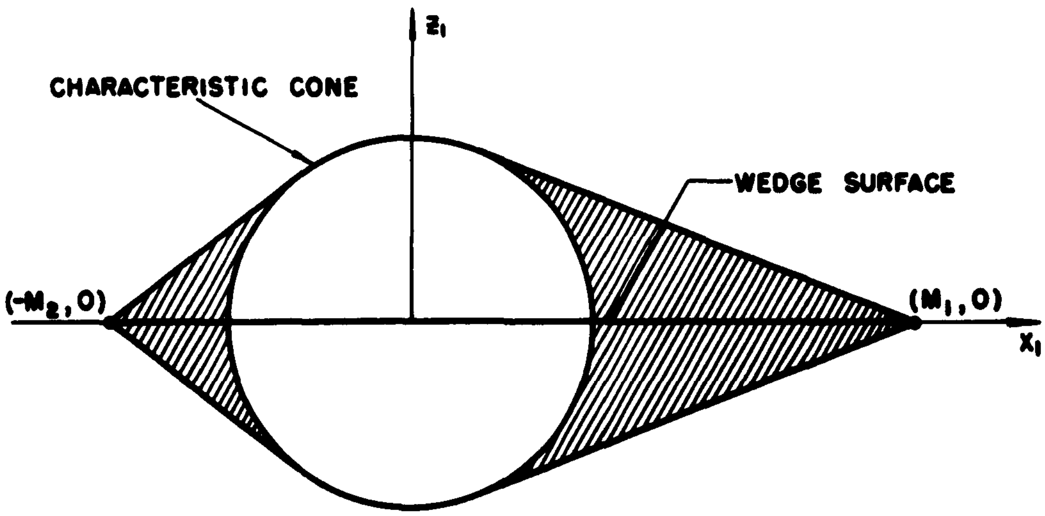


FIG. 6 SUPERSONIC CASE IN THE REDUCED COORDINATES PLANE

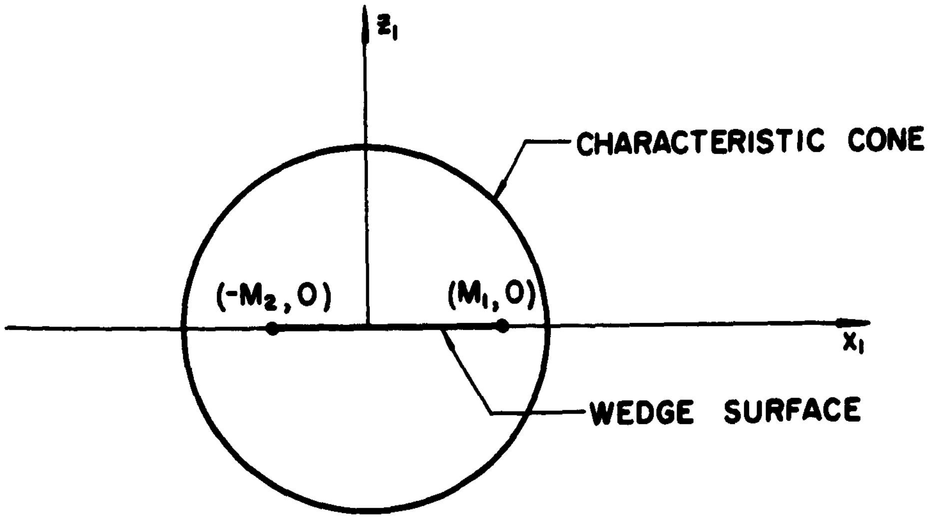


FIG. 7 SUBSONIC CASE IN THE REDUCED COORDINATES PLANE

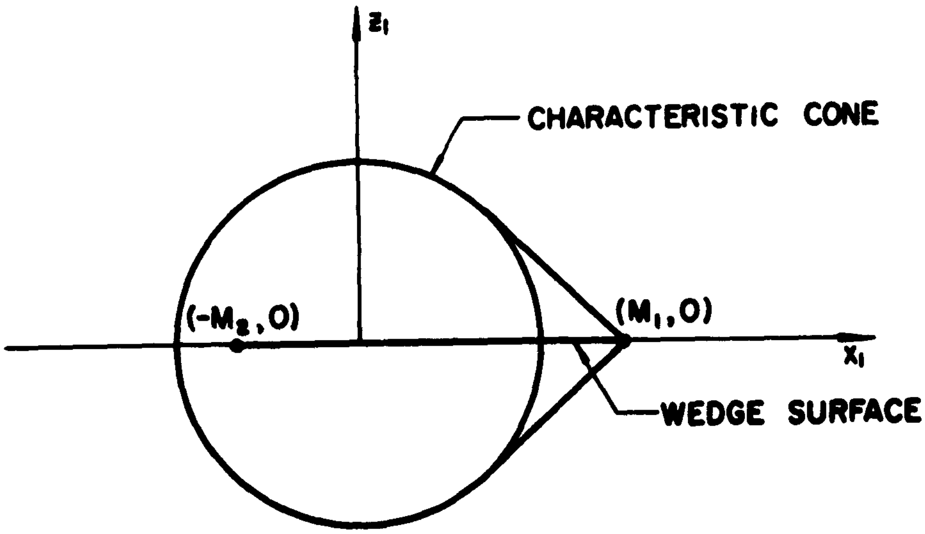


FIG. 8 MIXED CASE IN THE REDUCED COORDINATES PLANE

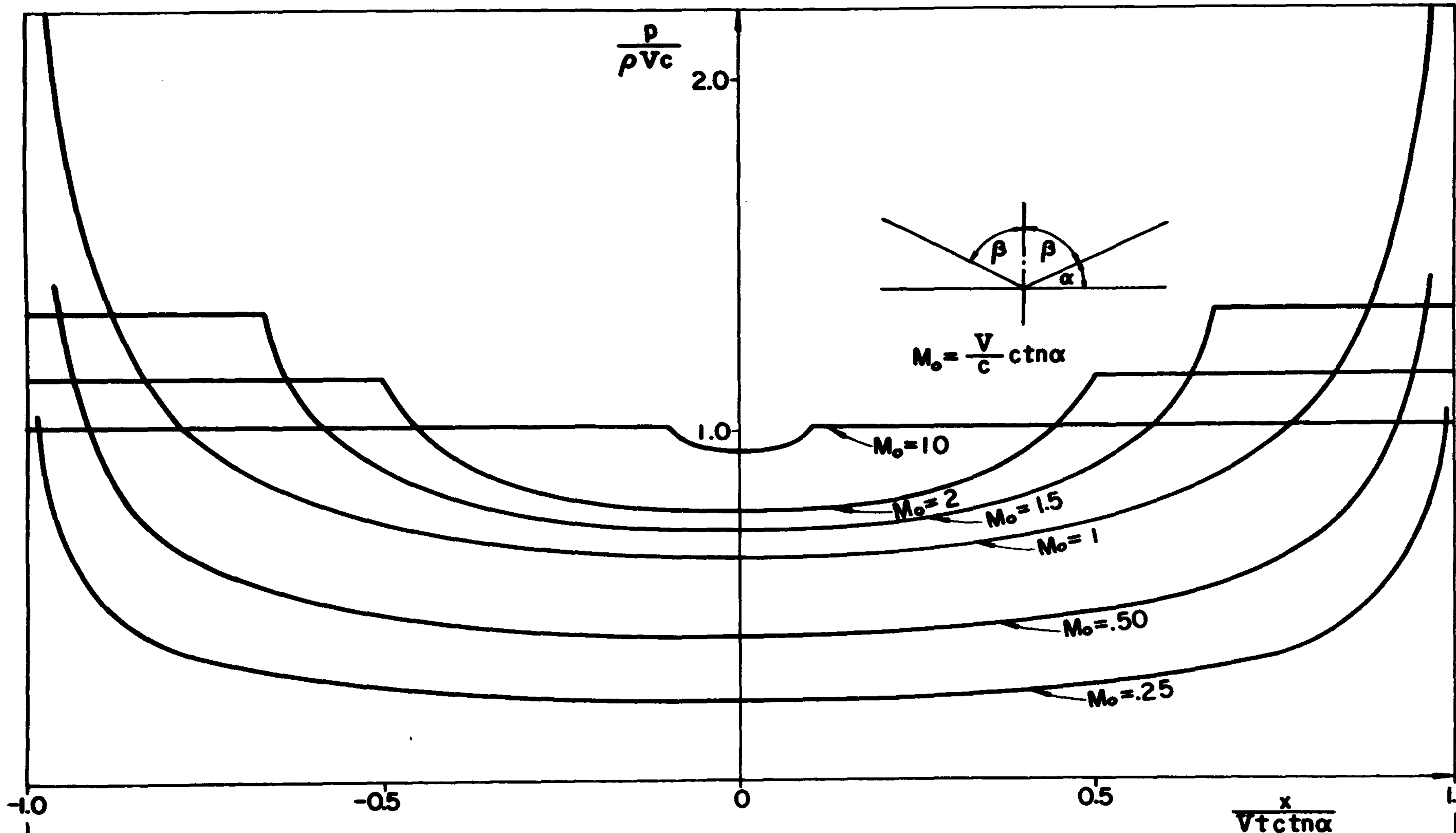


FIG. 9 PRESSURE FOR SYMMETRIC WEDGE ENTRY CASES

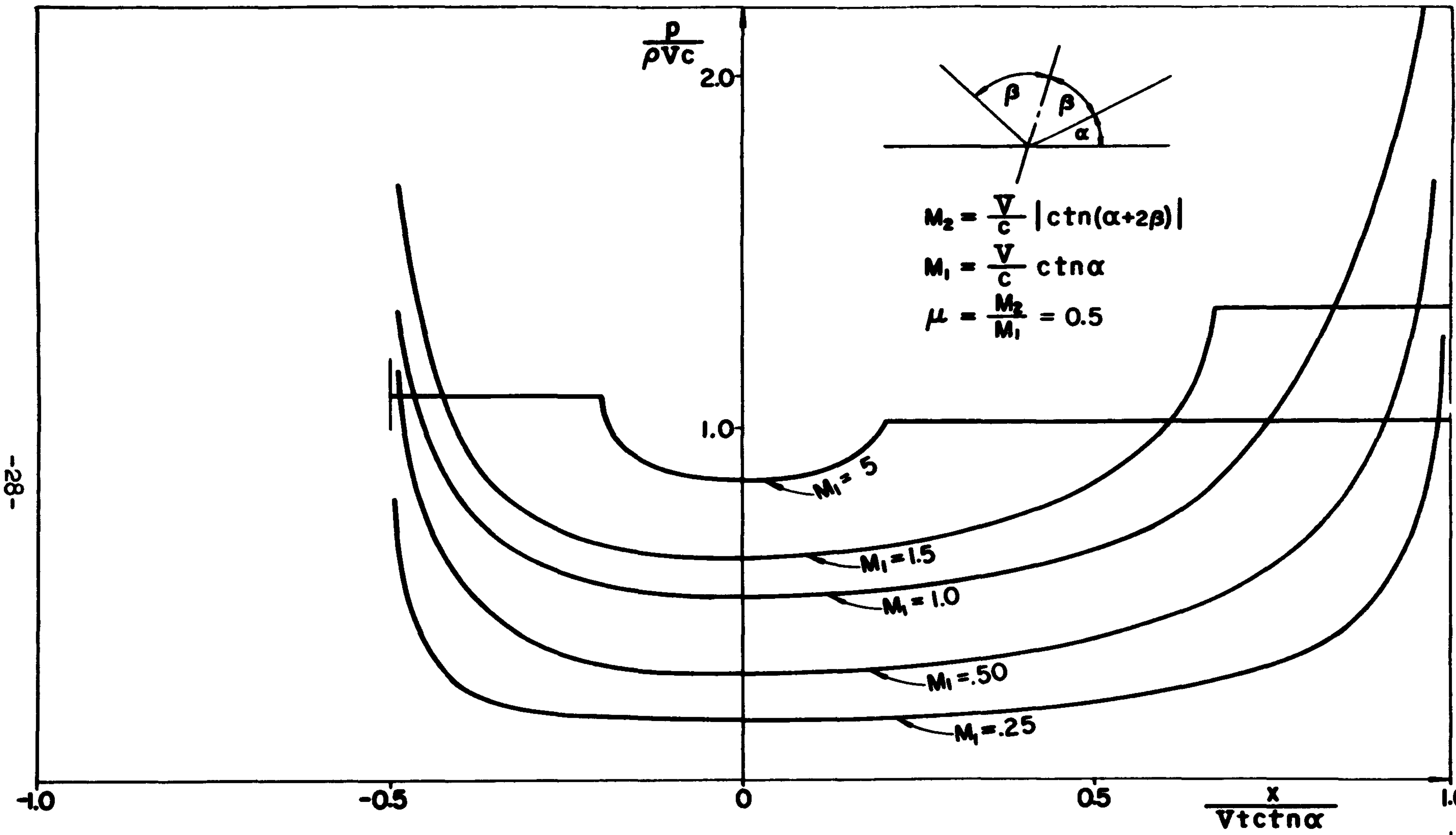


FIG. 10 PRESSURE FOR NON-SYMMETRIC WEDGE ENTRY CASES

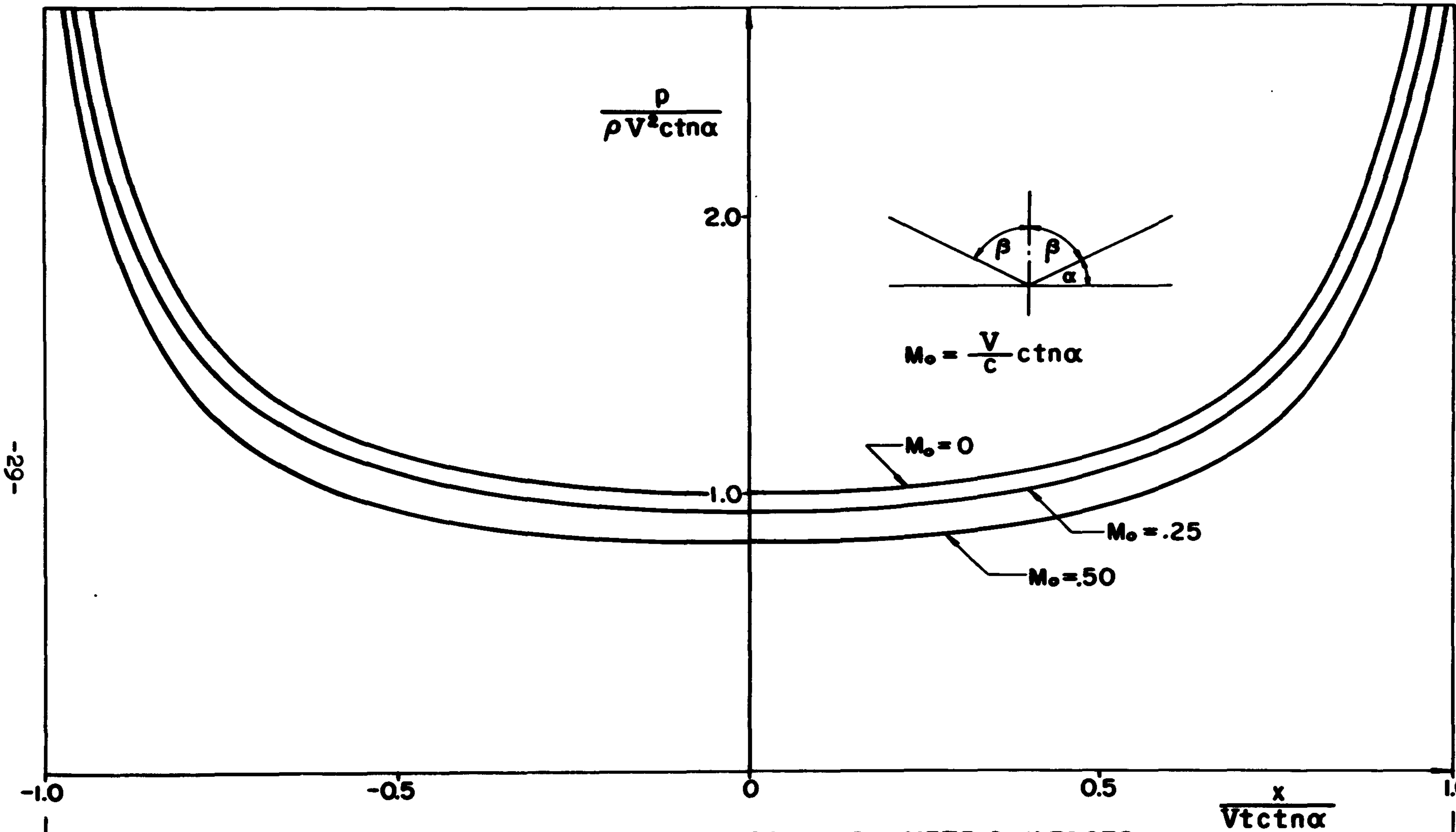


FIG. 11 PRESSURE ON SUBSONIC SYMMETRIC WEDGES